Int. J. Heat Mass Transfer. Vol. 8, pp. 297–301. Pergamon Press 1965. Printed in Great Britain

THE ROLE OF SHEAR AND FORM FORCES IN THE STABILITY OF A DRY PATCH IN TWO-PHASE FILM FLOW

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(*Received* 10 *July* 1964 *and in revised form* 28 *September* 1964)

Abstract—The Hartley–Murgatroyd force criterion for the stability of a dry patch is modified as a consequence of recent experimental results. The role of shear and form forces is examined and it is shown that the experimental results can be explained in a self-consistent manner by assuming that contact angles measured under static conditions are applicable to the flowing film. In the cases analysed the hitherto neglected shear and form forces appear to dominate the situation.

THE STABILITY of the dry patches which can form when thin films flow over solid surfaces is of fundamental importance to the "burnout" or "dry-out" which can occur when heat transfer is associated with this type of flow. In [1, 2] two stability criteria were suggested, one of which, the so-called force criterion involved the contact-angle between the film and the surface. The authors noted the lack of data which included contact angle measurements and stressed the need for more information.

Recently Hewitt and Lacey [3, 4] have reported experiments on dry patch stability in which they also measured static contact angles. In order to obtain agreement with the force criterion a contact angle of 17° had to be assumed, whereas the measured static contact angle was 49 ± 4°. The use of the measured contact angle resulted in a discrepancy of about 8:1 in the magnitude of the dewetting force. In the present note the effects of other forces are considered, and it is concluded that these could reasonably account for the observed discrepancy. It is therefore unnecessary to postulate a large difference between measured static contact angles and those existing at the edge of the dry patch. However, the extra forces are, at present, known so imprecisely that one should not conclude that the two contact angles are identical.

Figure 1(a) shows the upstream part of a dry patch which has already formed, together with a few approximate stream surfaces. The particular surface EG which passes through the stagnation point G is shown in section at 1(b). The point E (distant l from G) is assumed to be sufficiently far upstream for the flow at E to be unperturbed by the dry patch. The thickness of the film at E is δ, and downstream of E it is assumed to remain of order of magnitude δ. Consider the infinitesimally thin element of liquid centred on EG and shown shaded in Fig. 1(a). The following forces act on this element:

(*a*) *Shear forces*

Upstream of E the shear stress τ_{fg} from the gas phase is balanced by the stress τ_{sf} at the solid/liquid interface. The film velocity u and, therefore, τ_{sf} $(\equiv \mu_f \partial u/\partial y)_{y=0}$ decrease to zero at G, whereas τ_{fg} is assumed constant except very close to G. Thus the out-of-balance shear force on the element equals

$$l\tau_{fg} - \int_0^l \tau_{sf}\,\mathrm{d}x \equiv \lambda\,\tau_{fg}\ \text{(say)}$$

where this identity is used to define a length λ, such that $\lambda\tau_{fg}$ measures the out-of-balance shear force, as shown in Fig. 1(c). Now we are considering a state of affairs in which a dry patch is so large that surface tension forces in the plane of the solid surface are negligible, i.e. $R \gg \delta$ where R is a dimension typical of the dry patch. In practice R/δ probably lies in the range 10^1–10^2. In these circumstances one can apply an order-of-magnitude analysis to the

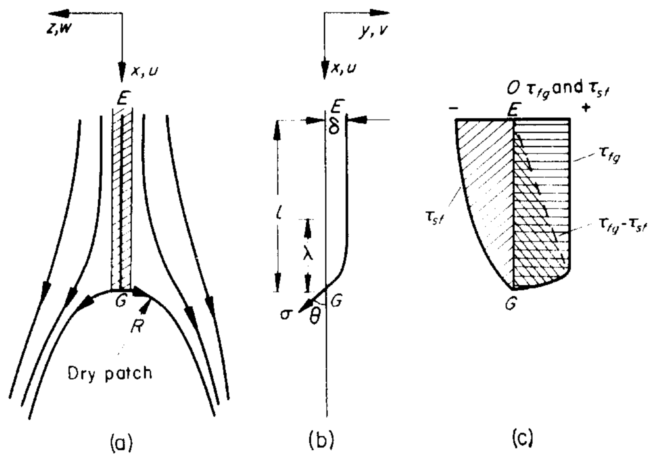

FIG. 1.

Navier–Stokes equations and show that, in our element EG (except for a small region near G) $\partial v/\partial y \ll (\partial u/\partial x$ or $\partial w/\partial z)$. It is clear, therefore, from the general shape of streamlines upstream of a two dimensional bluff body, that λ is of the same order of magnitude as R. We therefore write $\lambda = k_1\delta$, and represent the shear forces by a term $k_1\tau_{fg}\delta$ where k_1 probably lies in the range 10^1–10^2.

(*b*) *Form forces*

The region of the film near G represents a disturbing step to the gas flow, which will result in a small change in the static pressure around the curved surface of the film. Moreover rough experiments carried out by Hartley of this laboratory using a gravity motivated film indicate that a standing wave pattern or a "piling up" can occur upstream of G. As a consequence of these effects the normal pressure at the liquid/gas interface is non-uniform and the resulting "form force" has a downstream component which we shall call F. In order to estimate the order of magnitude of F we may note the correlation by Chung [5] of the data by Edwards and Sherriff [6] for the pressure drop due to small transverse wires attached to a surface. If the wires are far enough apart, the force per unit length on each wire is equal to 60 τd where τ is the shear stress at the undisturbed surface and d is the wire diameter. Close spacing of the wires results in a smaller force per wire. We identify d with the typical film thickness δ and tentatively write $F = k_2\tau_{fg}\delta$.

In the case of a film edge as shown in Fig. 1(b), i.e. without standing waves or "piling up" one would expect k_2 to be appreciably less than the value 60, which applies to a wire.

This way of allowing for the form drag receives further support from results by Wieghardt [7] for flow over a two dimensional step. These have been re-analysed in a manner more suitable for plausible extrapolation, as shown in the appendix. The results indicate that k_2 lies in the range 15–25, which is consistent with it being less than the Edwards and Sherriff data for a cylinder. One should not expect either set of data to yield a close estimate for k_2, since in the present situation three-dimensional conditions obtain.

(*c*) *Surface forces*

These are discussed by Hartley and Murgatroyd [1, 2] and amount to an upstream force equal to $\sigma(1 - \cos\theta)$.

(*d*) *Momentum flux*

The flux of x-momentum across unit width of film at E is $\int_0^\delta \rho_f u_0^2 \, dy$ where $u_0(y)$ is the velocity in the undisturbed film at E. If we assume that EG is a straight line and that flow is symmetrical about it, it can be shown, by expanding u and w in Taylor series about their values on the line EG, that the ratio of the flow of x-momentum from the element due to the w component to the flow of x-momentum due to u vanishes, in the limit, with δx. Thus in the limit we need only consider the term $\int_0^\delta \rho_f u_0^2 \, dy$.

The momentum equation is thus

$$\tfrac{1}{2} \int_0^\delta \rho_f u_0^2 \, dy + k \tau_{fg} \delta = \sigma (1 - \cos \theta) \qquad (1)$$

with

$$k \equiv k_1 + k_2$$

in which k_1 and k_2 (and therefore k) can be expected to be in the range of magnitude 10^1–10^2.

Since we are neglecting the effect of body-forces and waves on the velocity profile it follows that

$$u_0 \left(\frac{\rho_f}{\tau_{fg}}\right)^{\frac{1}{2}} = f \left[\frac{y}{\nu_f} \left(\frac{\tau_{fg}}{\rho_f}\right)^{\frac{1}{2}}\right]$$

or, in the familiar nomenclature:

$$U^+ = f(y^+), \text{ where } U^+ \equiv u_0/U^*, \; U^* \equiv (\tau_{fg}/\rho_f)^{\frac{1}{2}}$$

$$y^+ \equiv y U^*/\nu_f$$

Inserting these relations into equation (1) we have

$$\mu_f U^* \left[\tfrac{1}{2} I_2(\delta^+) + k\delta^+\right] = \sigma (1 - \cos \theta) \qquad (2)$$

where $I_2 \equiv \int_0^{\delta^+} U^{+^2} \, dy^+$ and $\delta^+ \equiv U^* \delta/\nu_f$

from which it is clear that the ratio of the film momentum to the shear and wake forces is $I_2/2k\delta^+$.

We now assume that the von Kármán velocity profile is applicable. I_2 has been tabulated for this profile in references [1, 2]. For normally occurring values of δ^+, say $\delta^+ < 100$, it turns out that $I_2/2\delta^+ < 125$. Since from the arguments above we might expect k to lie between 10 and 100, it would be unwise to neglect *a priori* any of the three forces.

Comparison with experimental results

The measurements by Hewitt and Lacey have already been referred to. They have been re-interpreted in the light of equation (2), taking their measured contact angles and using equation (2) to obtain an estimate of the factor k which accounts for shear and form forces. The results are given in Table 1. In this table the results from the last few runs should be treated with reserve, since in these cases the regime boundaries are being approached.

Table 1. Analysis of Hewitt and Lacey's experiments (assuming $\theta = 49^0$)

Run	U^*	δ^+	$\frac{1}{2}I_2$	$k\delta^+$	k	Estimate† of k_2
10	0·164	5·17	6·65	383	74	12·1
20	0·174	6·87	11·75	356	51·9	18·1
27	0·189	6·87	11·75	327	47·6	21·1
28	0·209	6·51	10·55	295	45·3	22·2
36	0·216	6·49	10·55	283	43·8	20·2
47	0·221	5·1	6·5	282	55·5	23·9
58	0·272	5·1	6·5	228	44·8	22·0
68	0·305	5·63	7·88	202	35·9	23·8
76	0·366	5·17	6·65	169	32·7	24·2
84	0·374	5·51	7·56	164	29·8	24·0
92	0·402	5·1	6·5	153	30·0	24·0
101	0·443	4·62	5·35	139	30·0	23·0

† Based on theory presented in Appendix.

The hitherto neglected interfacial forces characterized by k_1 and k_2 certainly exist but we have no direct evidence of their magnitude. The Edwards and Sheriff data suggest that $k_2 < 60$ and the extrapolation of Wieghardt's data suggests that $12 < k_2 < 25$, which is consistent. Moreover although there is no direct evidence as to whether or not the contact angle measured under static conditions is applicable to flowing films, if we assume it to be so Table 1 shows that k_1 lies in the range 10–50 (if we regard the last few runs with suspicion as recommended by Hewitt and Lacey). This agrees with the order-of-magnitude estimate made above, that $10^1 < k_1 < 10^2$. Thus a self consistent model is obtained.

This assumption about the contact angle also implies that, over the range of these experiments, the interfacial forces are dominant, since they are some twenty times as great as the film momentum terms. This can be seen by comparing columns 4 and 5. Equation (1) thus reduces to the very simple criterion

$$k\tau_{fg}\delta = \sigma(1 - \cos\theta) \quad (3)$$

Unfortunately there is at present insufficient accurate data to attempt to relate k_1 and k_2 to the flow conditions with any useful degree of precision.

ACKNOWLEDGEMENTS

I am indebted to Mr. Chung for drawing my attention to ref. (7) and to Mr. D. E. Hartley for stimulating discussions, for evaluating C_w' and presenting equation (A.1).

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APPENDIX: THE USE OF WIEGHARDT'S DATA

Reference [7] reports measurements of the extra force drag on a flat plate in a turbulent air stream due to a step in the plate having dimensions small compared with the local boundary layer thickness. The results are expressed in terms of a friction factor C_w based on the step area and the free stream dynamic head, a local Reynolds number based on step height, and the ratio of step height to local boundary layer thickness. The results have now been re-analysed in terms of a friction factor C_w' based on the average local dynamic head in the boundary layer up to the height of the step. The two quantities C_w and C_w' are shown in Table A.1.

Table A.1. Drag on a step in a flat plate
(A plot of Weighardt's data)

Downsteam position of step) (m)	Step height (cm)	$\log_{10} Re$ based on step height	Ratio of step height to thickness of boundary layer	Drag coeff. C_w based on free stream dyn. head	Drag coeff. C_w based on local average dyn. head
4·39	0·244	3·6	0·036	0·06	0·20
4·39	0·508	3·92	0·076	0·07	0·19
4·39	0·966	4·2	0·144	0·08	0·18
4·39	1·53	4·4	0·228	0·11	0·21
1·68	0·244	3·6	0·076	0·09	0·24
1·68	0·508	3·92	0·159	0·09	0·20
1·68	0·966	4·2	0·302	0·11	0·20
1·68	1·53	4·4	0·48	0·12	0·19

Within the range observed, C'_w turns out to be approximately constant, whereas C_w varies considerably.

This data has been applied to the results of reference [3] by assuming that C'_w is a constant equal to 0·2 for the lower values of step-to-boundary layer thickness obtaining in reference [3]. Assuming a $(1/7)^{th}$ power low for the gas velocity the parameter k_2 can be expressed in terms of the quantities tabulated in reference [3] thus:

$$k_2 = \frac{0{\cdot}2}{F_2} \times \frac{64}{63}\left(\frac{2m}{d_{e_2}}\right)^{2/7} \qquad \text{(A.1)}$$

where m = film thickness

d_{e_2} = effective hydraulic diameter of outer part of annulus.

These values of k_2 are the ones shown in Table 1.

Zusammenfassung—Das Hartley–Murgatroyd-Kraft-Kriterium für die Stabilität einer Trockenstelle wird als Folge jüngster experimenteller Ergebnisse modifiziert. Der Einfluss der Scher- und Formkräfte wird überprüft. Es wird gezeigt, dass die Versuchsergebnisse mit einer in sich konsistenten Art und Weise durch die Annahme erklärt werden können, dass die unter statischen Bedingungen gemessenen Randwinkel auf den strömenden Film angewendet werden können. In den analysierten Fällen scheinen die bisher vernachlässigten Scher- und Formkräfte die Situation zu beherrschen.

Аннотация—В результате полученных недавно экспериментальных результатов видоизменен критерий силы Хартли-Мургатройда для стабильности сухого участка. В статье исследована роль сил трения и форм-сил и показано, что экспериментальные результаты можно объяснить, самосогласованно допуская использование углов контакта, измеренных при статических условиях, в случае движущейся пленки. Оказывается, что силы трения и форм-силы, которыми до сих пор пренебрегали, играют доминирующую роль в анализируемых случаях.